1. (i)

E(Y3|D = 1) – E(Y3|D=0) = beta

Y3 = alpha + beta\*D + error

Var(e) = sigma^2 (homo)

Var(beta\_hat) = sigma^2/(summation Di-D\_bar) = sigma^2/(N\*D\_bar(1-D\_bar))

Var(e|D=1) = sigma1^2

Var(e|D=0) = sigma0^2 (heter)

Var(beta\_hat) = (summation Di(Di-D\_bar)^2)/(summation (Di-D\_bar)^2)^2 \* Var(e|D=1) +

= (summation(1 – Di)\*(Di-D\_bar)^2)/(summation (Di-D\_bar)^2)^2 \* Var(e|D=0)

B\_hat = 1/N summation DiYi - 1/N summation (1-Di)Yi

E(Y | D=1) E(Y | D=0)

Var(beta\_hat) = sigam1^2/N1 + sigma0^2/N0

(ii) Y13 = Y3 which replaces CEO Y03 = Y3 which doesnt

Causal effect = Y13 – Y03 ATE = E(Y13-Y03)

Y3 = D\*Y13 + (1-D)\*Y03

Assume Y03 Y13 independent to D (randomly assigned new CEO, which is unlikely)

Or D and error is independent (same thing)

Then E(Y03|D=1) = E(Y03|D=0) and E(Y13|D=1) = E(Y13|D=0)

Beta = E(Y3|D=1)-E(Y3|D=0)

= E(Y13|D=1)-E(Y03|D=0)

= E(Y13)-E(Y03) = E(Y13 – Y03) = ATE

Y2 = Y02, Y1 = Y01, if assumption is correct, then

E(Y2)=E(Y02)

E(Y2 | D=1 ) = E(Y02 | D = 1)

E(Y2 | D=0) = E(Y02 | D=0)

Check the difference of these two, not significantly differ from 0 then assumption is correct(this check is not perfectly effective)

(iii)

Y03 – Y02 is independent to D (assumption for dif-in-dif)

E(Y03 – Y02 | D=1) = E(Y03 – Y02 | D=0)

ATET = E(Y13 – Y03 | D=1) = dif-in-dif estimator

(iv)

T = year dummy 0 if year 2 1 if year 3

D = treatment dummy as above

Y =alpha + gamma1 \* D + gamma2 \* T + beta \* D\*T +error

Homo case:

Var (e | D= d, T=t) where d = 0,1; t=0, 1 (4 groups)

= (sigma \_dt)^2

Beta\_hat = (Y11\_bar – Y01\_bar) – (Y10\_bar – Y00\_bar)

Var(beta\_hat) = Var(Y11)/N11 + Var(Y10)/N10 +… + … (where all Var = sigma^2 since homo)

(v)

E(Y03 – Y02 | D=1) = E(Y03 – Y02 | D=0)

If suspect is true, then E(Y03 – Y02 | D=1) < E(Y03 – Y02 | D=0), Dif-in-dif assumption does not hold

Could be biased estimator

(vi)

Calculate if E(Y2-Y1 | D=0) < E(Y2-Y1 | D=1)

(vii)

E(Y03 – Y02 | D=1) not = E(Y03 – Y02 | D=0)

Use conditionally dif-in-dif

Then assume E(Y03 – Y02 | Y1, Y2, D=1) = E(Y03 – Y02 | Y1, Y2, D=0)

(viii)

Y3 – Y2 = alpha + beta\*D + +gamma1\*Y1 + gamma2\*Y2 + error

Beta = E(Y3 – Y2 | D=1) – E(Y3-Y2 | D=0)

Problem 2

(i)

D = 1 selected

D = 0 not selected

Y = DY1 + (1-D) Y0

Assume D independent to Y0 Y1

T = 1 treated = 0 non treated

T independent to Y1 Y0 , E(error | T) = 0

Y = alpha + beta\* T + error

Use D as an IV for T if T is correlated to error

Reg T on D and predict T and plug in reg of Y on T (IV procedures)

E(Y | D=1) – E(Y| D=0)

= Alpha + beta\*E(T | D=1) - (alpha + beta\*E(T | D=0)

Solve for beta :

Beta = [E(Y | D=1) – E(Y| D=0)] / [beta\*E(T | D=1) - beta\*E(T | D=0)]